Derivation on the Breaking of an Elbow with an Arbitrary Angle at an Arbitrary Angle, rev 1.

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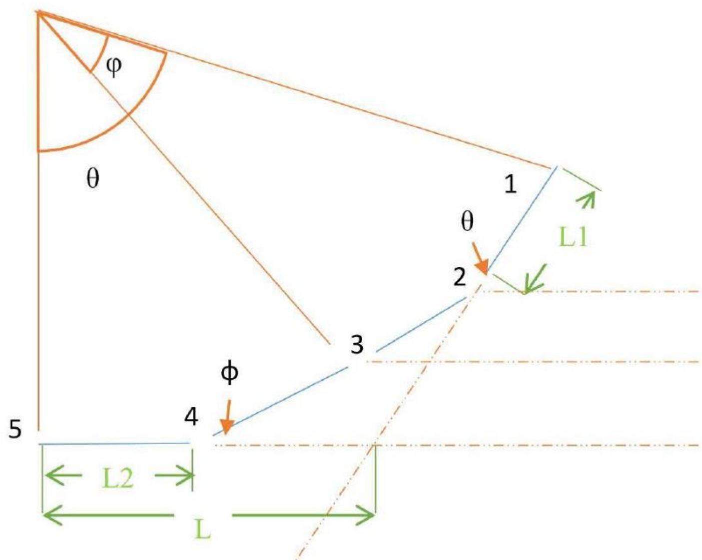

Given:

Bend 1-5 of Radius R, Bend Angle θ, to be split into two elbows (1-3 and 3-5) at angle ϕ for analysis.

The length from center of bend to end of bend. (Note: L to be substituted in 45° bends to match ASME B16.9 as needed.)

$$L = R \cdot tan\left(\frac{\theta}{2}\right)$$

Sum the horizontal components:

$$L_{12}cos(\theta) + L_{23}cos(\varphi) + L_{34}cos(\varphi) + L_{45} = L + Lcos(\theta)$$

Sum the vertical components:

$$L_{12}sin(\theta) + L_{23}sin(\varphi) + L_{34}sin(\varphi) = Lsin(\theta)$$

Remembering that in CAESAR:

$$L_{12} = L_{23} = L_1, L_{34} = L_{45} = L_2$$

Simplifying

Horizontal

$$L_1cos(\theta) + L_1cos(\varphi) + L_2cos(\varphi) + L_2 = L + Lcos(\theta)$$

$$L_1\big(cos(\theta) + cos(\varphi)\big) + L_2(cos(\varphi) + 1) = L(cos(\theta) + 1)$$

And Vertical

$$L_1 sin(\theta) + L_1 sin(\varphi) + L_2 sin(\varphi) = L sin(\theta)$$

$$L_1\big(sin(\theta) + sin(\varphi)\big) + L_2 sin(\varphi) = L sin(\theta)$$

And solving for L1 from the horizontal components:

$$L_1 = \frac{L(cos(\theta) + 1) - L_2(cos(\varphi) + 1)}{cos(\theta) + cos(\varphi)}$$

Substituting in the horizontal L1 reduction into the vertical equation:

$$\frac{L(cos(\theta) + 1) - L_2(cos(\varphi) + 1)}{cos(\theta) + cos(\varphi)}\big(sin(\theta) + sin(\varphi)\big) + L_2 sin(\varphi) = L sin(\theta)$$

Expanding the left side of the equation:

$$L(cos(\theta) + 1)\frac{sin(\theta) + sin(\varphi)}{cos(\theta) + cos(\varphi)} - L_2(cos(\varphi) + 1)\frac{sin(\theta) + sin(\varphi)}{cos(\theta) + cos(\varphi)} + L_2 sin(\varphi) = L sin(\theta)$$

Separating out L2 to one side of the equation:

$$L_2 sin(\varphi) - L_2(cos(\varphi) + 1)\frac{sin(\theta) + sin(\varphi)}{cos(\theta) + cos(\varphi)} = L sin(\theta) - L(cos(\theta) + 1)\frac{sin(\theta) + sin(\varphi)}{cos(\theta) + cos(\varphi)}$$

Simplifying the right side of the equation:

$$L_2\left(sin(\varphi) - (cos(\varphi) + 1)\frac{sin(\theta) + sin(\varphi)}{cos(\theta) + cos(\varphi)}\right) = L\left(sin(\theta) - (cos(\theta) + 1)\frac{sin(\theta) + sin(\varphi)}{cos(\theta) + cos(\varphi)}\right)$$

Solving for L2.

$$L_2 = L\frac{\left(sin(\theta) - (cos(\theta) + 1)\frac{sin(\theta) + sin(\varphi)}{cos(\theta) + cos(\varphi)}\right)}{\left(sin(\varphi) - (cos(\varphi) + 1)\frac{sin(\theta) + sin(\varphi)}{cos(\theta) + cos(\varphi)}\right)}$$

Notes:

- Bends shall be specified at points 2 and 4 only. The radius of both bends is R.
- L12 and L45 must be at least this long. They may be longer, but any excess of this value will be straight piping.
- An error message may appear as you apply a bend at these locations if you specify minimum lengths, and CAESAR automatically attempts to place a node at this location when you apply the bend modifier.
- The mathematics above assumes that L45 is oriented along a primary axis. It may be necessary to temporarily re-orient the entire model or sections of the model in order to work the math out.
- Only one break is considered, but the logic can be extended to multiple breaks. For example, instead of a 30° bend broken at 10° and 20°, you can think of it as two 15° bends broken at 10° and 5°. The second 15° bend must, however, be rotated 15° relative to the first 15° bend to act as a continuous 30° bend.